THE ANGULAR WIDTH OF THE COHERENT BACK-SCATTER OPPOSITION EFFECT: AN APPLICATION TO ICY OUTER PLANET SATELLITES

(Letter to the Editor)

M. I. MISHCHENKO

Main Astronomical Observatory of the Ukrainian Academy of Sciences, Kiev, Goloseevo, Ukraine

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Abstract. It has been suggested recently that coherent back-scattering of light from powder-like regolithic surfaces can explain remarkable opposition brightening of some atmosphereless solar system bodies. In this paper, a dense-medium light-scattering theory is used to calculate the half-width at half-maximum (HWHM) of the coherent back-scattering peak for a number of scattering models. We demonstrate that HWHM strongly depends on the optical properties of the scattering medium and can serve as a critical test in comparing alternative models. It is shown that coherent back-scattering may be a likely explanation of the opposition effect exhibited by icy outer planet satellites.

1. Introduction

Recently, it has been suggested that coherent back-scattering of light (CBL) from discrete random media (or weak localization of photons) can account for a remarkable opposition brightening exhibited by outer planet satellites (Hapke, 1990; Domingue *et al.*, 1991) and Saturn's rings (Mishchenko and Dlugach, 1992). For media composed of randomly distributed scattering particles and illuminated by a parallel beam of light, CBL manifests itself as a well-defined narrow peak in the reflected light at phase angles near zero (e.g., Sheng, 1990; Nieto-Vesperinas and Dainty, 1990). One of the main characteristics of the coherent back-scattering peak is its half-width at half-maximum (HWHM). Interference nature of CBL leads to a fundamental result according to which for optically thick media HWHM is proportional to the ratio of the wavelength of light λ to the transport mean free path of photons in the medium λ_{tr}. For media consisting of lossless or slightly lossy scatterers, we obtain

$$\mathrm{HWHM} = \frac{\varepsilon\lambda}{2\pi\lambda_{tr}}, \tag{1}$$

where

$$\lambda_{tr}^{-1} = nC_{sca}(1 - \langle\cos\vartheta\rangle) \tag{2}$$

and ε is a constant close to 0.5 (Stephen and Cwilich, 1986; van der Mark *et al.*, 1988; Wolf *et al.*, 1988; Barabanenkov and Ozrin, 1988). In Equation (2), n is the number of particles per unit volume, C_{sca} is the scattering cross section, and $\langle\cos\vartheta\rangle$ is the mean cosine of the scattering angle. For sparsely distributed spherical scatterers, the quantities

C_{sca} and $\langle \cos\vartheta \rangle$ may be computed from the Mie theory (e.g., van de Hulst, 1957). However, regolithic grains in upper surface layers of atmosphereless bodies are likely to be densely packed rather than sparsely distributed. For such media, especialy for those consisting of particles smaller than the wavelength, spatial correlation among scatterers can lead to a substantial increase of λ_{tr} as compared with sparsely distributed particles. Therefore, more rigorous dense-medium scattering theories should be used to obtain reliable results.

It is the purpose of this paper to calculate HWHM for a number of scattering models by using a dense-medium theory, in which spatial correlation among scatterers is taken into account by introducing the so-called static structure factor (e.g., Twersky, 1983; Tsang and Kong, 1983; Wolf *et al.*, 1988; Saulnier *et al.*, 1990). In Section 2, basic definitions and formulae are recapitulated and the computational scheme for calculating HWHM is summarized. In Section 3, HWHM is calculated for a number of models of the scattering medium and dependence of HWHM on the optical characteristics of the medium is studied. Finally, in Section 4, an application to icy outer planet satellites is given.

2. Basic Definitions and Formulae

By definition, for a sparse, discrete, macroscopically isotropic medium,

$$C_{\text{sca}} = \int_{4\pi} \mathrm{d}\Omega \, \frac{\mathrm{d}C_{\text{sca}}}{\mathrm{d}\Omega} \,, \tag{3}$$

$$C_{\text{sca}} \langle \cos\vartheta \rangle = \int_{4\pi} \mathrm{d}\Omega \, \frac{\mathrm{d}C_{\text{sca}}}{\mathrm{d}\Omega} \cos\vartheta \,, \tag{4}$$

where $\mathrm{d}C_{\text{sca}}/\mathrm{d}\Omega$ is the differential scattering cross section and ϑ is the scattering angle (e.g., Bohren and Huffman, 1983). For densely packed media, spatial correlation among scattering particles can be taken into account by multiplying the differential scattering cross section by the static structure factor $S(\vartheta)$ (e.g., Balescu, 1975; Twersky, 1983; Tsang and Kong, 1983; Wolf *et al.*, 1988; Saulnier *et al.*, 1990). Thus, Equations (3) and (4) are replaced by the formulae

$$C_{\text{sca}} = \int_{4\pi} \mathrm{d}\Omega \, \frac{\mathrm{d}C_{\text{sca}}}{\mathrm{d}\Omega} \, S(\vartheta) \,, \tag{5}$$

$$C_{\text{sca}} \langle \cos\vartheta \rangle = \int_{4\pi} \mathrm{d}\Omega \, \frac{\mathrm{d}C_{\text{sca}}}{\mathrm{d}\Omega} \, S(\vartheta) \cos\vartheta \,. \tag{6}$$

The structure factor is given (Balescu, 1975) by

$$S(\vartheta) = 1/[1 - nC(p)] \,, \tag{7}$$

where $C(p)$ is the three-dimensional Fourier transform of the direct correlation function $C(r)$,

$$C(p) = \int \mathrm{d}\mathbf{r} \exp(-i\mathbf{pr})C(r) \tag{8}$$

and $p = [4\pi \sin(\vartheta/2)]/\lambda$. To calculate the direct correlation function, we use the so-called Percus–Yevick approximation, which implies that the scattering particles are hard, impenetrable, monodispersive spheres of a radius r_0 (e.g., Balescu, 1975). Thus, we have

$$C(r) = \begin{cases} -\alpha - \beta r^* - \delta(r^*)^3, & \text{for} \quad r < 2r_0, \\ 0, & \text{for} \quad r > 2r_0, \end{cases} \tag{9}$$

where

$$r^* = r/(2r_0), \tag{10}$$

$$\alpha = \frac{(1 + 2f)^2}{(1 - f)^4}, \tag{11}$$

$$\beta = -6f \frac{(1 + f/2)^2}{(1 - f)^4}, \tag{12}$$

$$\delta = \alpha f/2 \tag{13}$$

and

$$f = \tfrac{4}{3}\pi n r_0^3 \tag{14}$$

is the filling factor (i.e., the fraction of a volume occupied by the particles). If we insert Equation (9) into Equation (8), we have (cf. Tsang and Kong, 1983)

$$nC(p) = 24f \left\{ \frac{(\alpha + \beta + \delta)}{u^2} \cos u - \frac{(\alpha + 2\beta + 4\delta)}{u^3} \sin u - \right.$$

$$\left. - \frac{2(\beta + 6\delta)}{u^4} \cos u + \frac{2\beta}{u^4} + \frac{24\delta}{u^5} \sin u + \frac{24\delta}{u^6} (\cos u - 1) \right\}, \tag{15}$$

where $u = 2pr_0$. For the particular case of $p = 0$, we have from Equations (8) and (9)

$$nC(0) = 24f(-\alpha/3 - \beta/4 - \delta/6). \tag{16}$$

Thus, the numerical computation of HWHM involves the following steps. First, the structure factor $S(\vartheta)$ is calculated via Equations (7), (15), and (16) for a given particle radius r_0 and filling factor f. Next, the differential scattering cross-section $\mathrm{d}C_{sca}/\mathrm{d}\Omega$ is calculated via the Mie formulae (e.g., van de Hulst, 1957; Bohren and Huffman, 1983). Next, the integrals in Equations (5) and (6) are computed numerically by use of a quadrature formula. Next, the transport mean free path is calculated via the formula

$$\lambda_{\mathrm{tr}}^{-1} = C_{\mathrm{sca}}(1 - \langle \cos \vartheta \rangle) \frac{3f}{4\pi r_0^3} \tag{17}$$

(cf. Equations (2) and (14)). Finally, HWHM is calculated via Equation (1).

Finally, we note that, in our computations, the well-known ripple structure of monodisperse Mie cross sections (e.g., Bohren and Huffman, 1983) was suppressed by use of a narrow gamma distribution of particle radii (Hansen and Hovenier, 1974)

$$n(r) = \text{constant}\, r^{(1-3b)/b} \exp[-r/(ab)] \,, \tag{18}$$

with $a = r_0$ and $b = 0.04$.

3. Calculations and Discussion

In Figures 1–3, HWHM is plotted versus a dimensionless size parameter $y = r_0/\lambda$ for three real refractive indices $N = 1.31$, 1.45, and 1.6 and four values of the filling factor

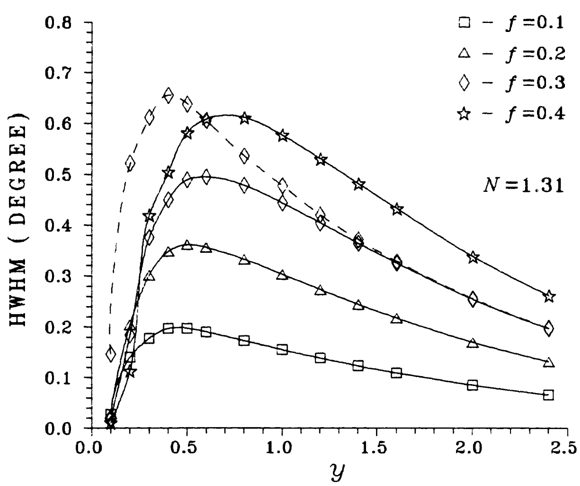

Fig. 1. HWHM versus a dimensionless size parameter $y = r_0/\lambda$ for the refractive index $N = 1.31$ and four values of the filing factor $f = 0.1$, 0.2, 0.3, and 0.4. The dashed curve is computed for $f = 0.3$ by use of sparse-medium formulae (3) and (4).

$f = 0.1$, 0.2, 0.3, and 0.4. The refractive index $N = 1.31$ is close to that of H_2O ice at the visible and near-infrared wavelengths (Warren, 1984), while the refractive indices $N = 1.45$ and 1.6 are characteristic for silicate materials. For the sake of comparison, in Figure 1 the dashed curve shows computations via sparse-medium formulae (3) and (4) for $N = 1.31$ and $f = 0.3$.

The following obvious properties of HWHM can be extracted from the data shown in Figures 1–3.

(i) The use of sparse-medium formulae (3) and (4) can lead to a substantial over-

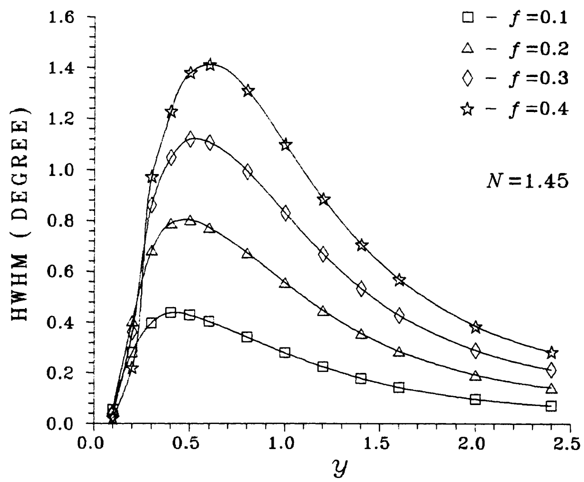

Fig. 2. As in Figure 1, for $N = 1.45$.

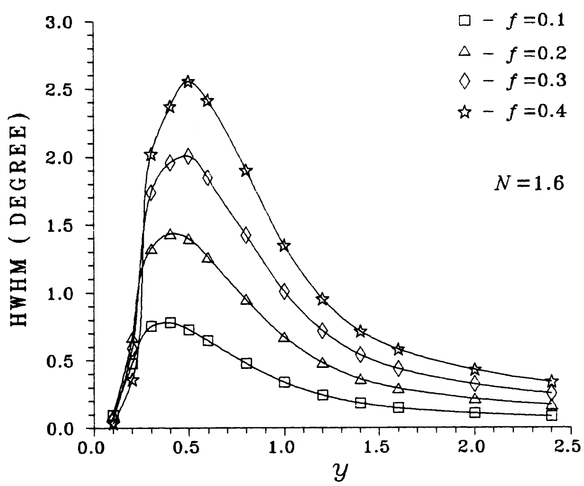

Fig. 3. As in Figure 1, for $N = 1.6$.

estimation of HWHM for $y < 1$. Therefore, these formulae should not be used for small size parameters (cf. Wolf *et al.*, 1988; Saulnier *et al.*, 1990). For larger size parameters ($y > 1$), the sparse-medium formulae give (very) good results and in the limit $y \to \infty$ the difference between the sparse- and dense-medium formulae vanishes.

(ii) HWHM tends to zero with both $y \to 0$ and $y \to \infty$. Therefore, the opposition brightening due to CBL by particles either much smaller or much larger than the wavelength cannot be observed if the range of very small phase angles is inaccessible.

(iii) All curves have a maximum near $y = 0.5$. With increasing filling factor, this maximum shifts towards greater size parameters.

(iv) The maximum is very sharp for large refractive indices and becomes (much) less sharp for small refractive indices.

(v) The value of the maximum is (much) greater for greater refractive indices.

4. An Application to Icy Outer Planet Satellites

In this paper, we have used a dense-medium scattering theory to calculate HWHM of the coherent back-scattering peak for a number of scattering models and have demonstrated that HWHM is strongly dependent on the optical properties of the scattering medium. In particular, it follows from our computations that for ice-covered surfaces, HWHM should be small (a few tenths of degree) and can be nearly wavelength-independent in a wide spectral region. This was really observed for Saturn's rings (Franklin and Cook, 1965), Uranian satellites (Brown and Cruikshank, 1983; Goguen *et al.*, 1989), and Europa (Domingue *et al.*, 1991). All these objects exhibit opposition spikes with HWHM of about 0.2–0.3° at visible wavelengths. Also, for Uranian satellites, HWHM is nearly wavelength-independent at a wide range of wavelengths from the visible (0.55 μm) to the near infrared (2.2 μm). In Figure 4, the theoretically

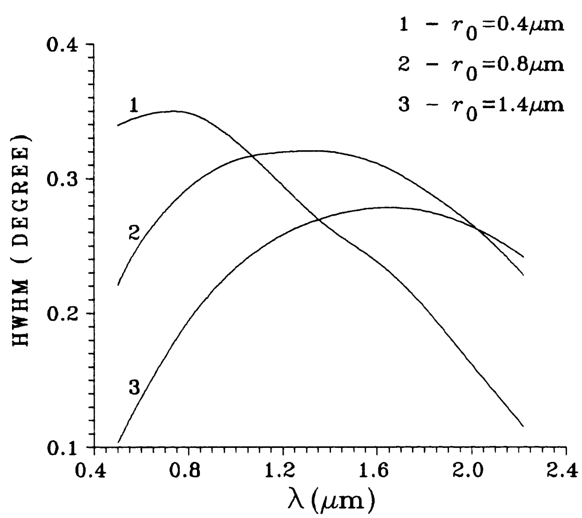

Fig. 4. HWHM versus wavelength for ice particles with $f = 0.2$ and $r_0 = 0.4$, 0.8, and 1.4 μm.

computed HWHM is plotted versus wavelength for ice particles with $f = 0.2$ and $r_0 = 0.4$, 0.8, and 1.4 μm. Spectral refractive indices of ice were taken from Warren (1984). One sees from Figures 1 and 4 that particles with radii of about 0.8 μm and filling factors of about 0.2 can reproduce the observed HWHM of 0.2–0.3° in this spectral range 0.55 to 2.2 μm. It is interesting to note that ice particles of essentially the same radii are known to be present in the outer B ring of Saturn and give rise to the so-called 'spokes' (e.g., Doyle *et al.*, 1989; Doyle and Grun, 1990). Also, Mishchenko and

Dlugach (1992) assumed that particles of Saturn's rings are covered with submicrometer-size ice grains and demonstrated that theoretical computations of the opposition effect produced by these grians via CBL are consistent with the observations of the opposition effect exhibited by Saturn's rings (Franklin and Cook, 1965). Therefore, we may suggest that submicrometer-size regolithic grains may be a common property of ice-covered airless surfaces at low temperatures. The possible origin of such grains is discussed, e.g., by Smoluchowski (1983).

Finally we note that for silicate surfaces, HWHM may be much greater (of the order of degree). Such HWHM of about 1° was observed for high-albedo asteroids 44 Nysa and 64 Angelina (Harris *et al.*, 1989). As follows from Figures 2 and 3, for silicate surfaces HWHM should be substantially wavelength-dependent. Therefore, meausement of the variation of HWHM with wavelength would test whether CBL is a likely explanation of the remarkable opposition spikes exhibited by these high-albedo asteroids (Mischchenko and Dlugach, 1991).

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